Coimisiún na Scrúduithe Stáit
State Examinations Commission

Scéimeanna Marcála *Scrúduithe Ardteistiméireachta, 2005*

Matamaitic Fheidhmeach *Gnáthleibhéal*

Marking Scheme *Leaving Certificate Examination, 2005*

Applied Mathematics *Ordinary Level*

General Guidelines

1 Penalties of three types are applied to candidates' work as follows:

Slips	- numerical slips	S(-1)
Blunders	- mathematical errors	B(-3)
Misreading	- if not serious	M(-1)

Serious blunder or omission or misreading which oversimplifies:
- award the attempt mark only.

Attempt marks are awarded as follows: 5 (att 2), 10 (att 3).

2 Mark all answers, including excess answers and repeated answers whether cancelled or not, and award the marks for the best answers.

3 Mark scripts in red unless candidate uses red. If a candidate uses red, mark the script in blue or black.

4 Number the grid on each script 1 to 9 in numerical order, not the order of answering.

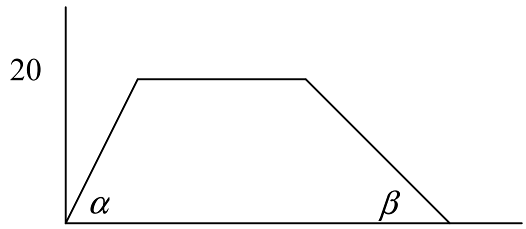
5 Scrutinise **all** pages of the answer book.

6 The marking scheme shows one correct solution to each question. In many cases there are other equally valid methods.

1. A particle travels from p to q in a straight line. It starts from rest at p and accelerates uniformly
to its maximum speed of 20 m/s in 10 seconds. The particle maintains this speed of 20 m/s for 15 seconds before decelerating uniformly to rest at q in a further 20 seconds.

(i) Draw a speed-time graph of the motion of the particle from p to q.
(ii) Find the uniform acceleration of the particle.
(iii) Find the uniform deceleration of the particle.
(iv) Find $|pq|$, the distance from p to q.
(v) Find the average speed of the particle as it moves from p to q, giving your answer in the form $\frac{a}{b}$, where $a, b \in \mathbf{N}$.

(i)



15

(ii)

$$v = u + at \qquad\qquad a = \tan\alpha$$

$$20 = 0 + 10a \qquad \text{or} \qquad a = \frac{20}{10}$$

$$a = 2 \qquad\qquad a = 2$$

10

(iii)

$$v = u + at \qquad\qquad a = \tan\beta$$

$$0 = 20 + 20a \qquad \text{or} \qquad a = \frac{20}{20}$$

$$a = -1 \qquad\qquad a = 1$$

deceleration is $1\ \text{m/s}^2$

10

(iv)

$$\text{distance} = \tfrac{1}{2}(10)(20) + (20)(15) + \tfrac{1}{2}(20)(20)$$
$$= 100 + 300 + 200$$
$$= 600$$

10

(v)

$$\text{average speed} = \frac{\text{total distance}}{\text{total time}}$$
$$= \frac{600}{45}$$
$$= \frac{40}{3}\ \text{m/s}$$

5

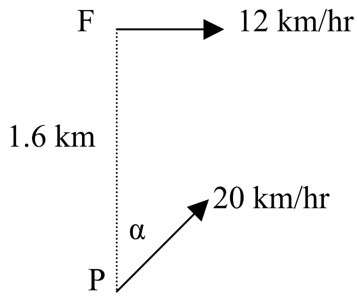
50

2. **(a)** Two athletes A and B are running due east in a race.
At a certain instant athlete A is x metres from the finishing line and is running with a constant speed of 8 m/s. At this instant athlete B is 6 metres behind A and is running with a constant speed of 10 m/s.
B catches up with A at the finishing line, so that the race ends in a dead heat.

(i) Find the velocity of B relative to A.

(ii) Find the value of x.

(b) A ferry F is travelling due east with a constant speed of 12 km/hr.
A boat P is travelling in the direction α degrees east of north with a constant speed of 20 km/hr.
At noon P is 1.6 km due south of F and t minutes later P intercepts F.



(i) Find the velocity of P relative to F, in terms of $\vec{i}$, $\vec{j}$ and α.

(ii) Find the value of α, correct to the nearest degree.

(iii) Find the value of t.

(a) (i)

$$V_{BA} = V_B - V_A$$ 5

$$= (10\,\vec{i}) - (8\,\vec{i})$$

$$= 2\,\vec{i}$$ 10

(ii)

$$\text{time} = \frac{6}{2}$$

$$= 3 \text{ s}$$ 5

$$s = ut + \tfrac{1}{2}at^2$$

$$x = 8(3) + 0$$

$$= 24 \text{ m}$$ 5

(b) (i)

$$V_{PF} = V_P - V_F$$ 5

$$= (20\sin\alpha\,\vec{i} + 20\cos\alpha\,\vec{j}) - (12\,\vec{i})$$ 10

(ii)

$$20\sin\alpha = 12$$

$$\sin\alpha = 0.6$$

$$\alpha = 37°$$ 5

(iii)

$$t = \frac{1.6}{20\cos\alpha}$$

$$= \frac{1.6}{16}$$

$$t = 0.1 \text{ h} \quad \text{or} \quad 6 \text{ minutes}$$ 5

50

3. **(a)** A particle is projected from a point o on level horizontal ground with an initial speed of $50\sqrt{3}$ m/s at an angle β to the horizontal.
It strikes the level ground at p after 15 seconds.

(i) Find the angle β.

(ii) Find $|op|$, the distance from o to p. Give your answer to the nearest metre.

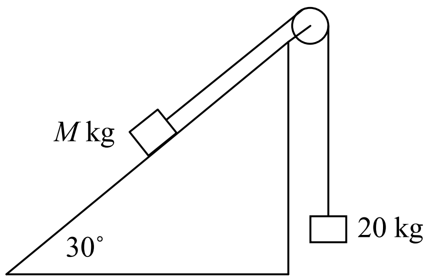
(b) A straight vertical cliff is 125 m high.
A projectile is fired horizontally with an initial speed of u m/s from the top of the cliff.
It strikes the level ground at a distance $375\sqrt{3}$ m from the foot of the cliff.

Find the value of u, correct to one decimal place.

(a) (i)	$s = ut + \frac{1}{2}at^2$	
	$0 = 50\sqrt{3}\sin\beta(15) + \frac{1}{2}(-10)(225)$	10
	$\sin\beta = \dfrac{75}{50\sqrt{3}}$ or $\dfrac{\sqrt{3}}{2}$	
	$\beta = 60°$	10
(ii)	$r_i = u(t)$	
	$= 50\sqrt{3}\cos\beta(15)$	
	$\lvert op\rvert = 50\sqrt{3}(\frac{1}{2})(15)$	
	$= 650$	10
(b)	$s_y = 0 - \frac{1}{2}(10)t^2$	
	$-125 = -\frac{1}{2}(10)t^2$	
	$t = 5$	10
	$s_x = ut$	
	$375\sqrt{3} = u(5)$	
	$u = 75\sqrt{3}$	
	$= 129.9$	10

50

4. A particle of mass M kg is placed on a rough plane inclined at 30° to the horizontal.
This particle is connected by a light inextensible string passing over a smooth light pulley at the top of the plane to a particle of mass 20 kg, hanging freely under gravity.
The coefficient of friction between the M kg mass and the plane is $\dfrac{2}{5\sqrt{3}}$.



The system is released from rest.
The 20 kg mass moves vertically upwards a distance of 16 m in 4 s.

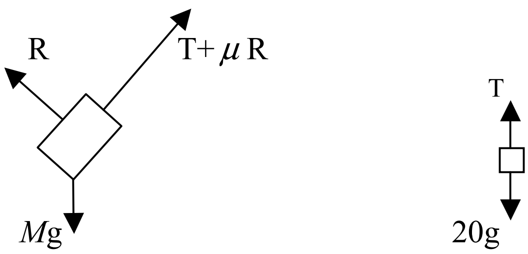
(i) Show on separate diagrams all the forces acting on each particle.

(ii) Show that the constant acceleration of the particles is 2 m/s^2.

(iii) Find the tension in the string.

(iv) Find the value of M.

(i)



10
10

(ii)

$$s = ut + \tfrac{1}{2}at^2$$

$$16 = 0 + \tfrac{1}{2}a(16)$$

$$a = 2$$ 5

(iii)

$$T - 20g = 20a$$ 5

$$T = 20(10) + 20(2)$$

$$= 240$$ 5

(iv)

$$Mg\sin 30 - \mu R - T = Ma$$ 5

$$R = Mg\cos 30$$ 5

$$Mg\left(\frac{1}{2}\right) - \mu(Mg\cos 30) - T = M(2)$$

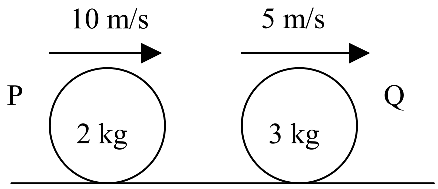
$$Mg\left(\frac{1}{2}\right) - \mu(Mg\cos 30) - 240 = M(2)$$

$$5M - 2M - 240 = M(2)$$

$$M = 240\text{ kg}$$ 5

50

5. A smooth sphere P, of mass 2 kg, moving with a speed of 10 m/s collides directly with a smooth sphere Q, of mass 3 kg, moving in the same direction with a speed of 5 m/s on a smooth horizontal table.



The coefficient of restitution for the collision is e.

After the collision, sphere Q continues to travel in the same direction but with a speed of 8 m/s.

(i) Find the speed of P after the collision.

(ii) Find the value of e.

(iii) Find the fraction of kinetic energy lost due to the collision.

(iv) Find the magnitude of the impulse imparted to each sphere.

(i) PCM $\quad 2(10) + 3(5) = 2v_1 + 3(8)$

$v_1 = 5.5$ 10

(ii) NEL $\quad v_1 - v_2 = -e(u_1 - u_2)$

$5.5 - 8 = -e(10-5)$

$e = \frac{1}{2}$ 10

(iii) KE before collision $= \frac{1}{2}(2)(10)^2 + \frac{1}{2}(3)(5)^2$

$= 137.5$ 5

KE after collision $= \frac{1}{2}(2)(5.5)^2 + \frac{1}{2}(3)(8)^2$

$= 126.25$ 5

KE lost $= 137.5 - 126.25$

$= 11.25$ 5

Fraction of KE lost $= \frac{11.25}{137.5} = \frac{9}{110}$ 5

(iv) Impulse $= (3)(8) - (3)(5)$

$= 9$ 10

50

6. **(a)** Particles of weight 3 N, 4 N, 1 N and 5 N are placed at the points $(-x, -3)$, $(2, y)$, $(1, 3)$ and (x, y), respectively.
The centre of gravity of the four particles is at the origin.

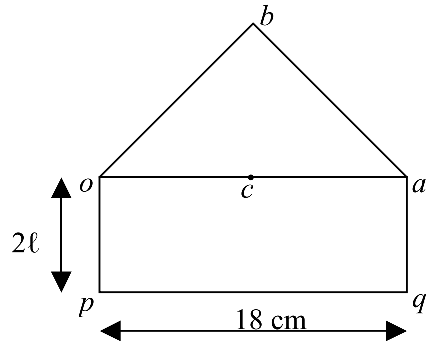
Find the value of x and the value of y.

(b) A uniform lamina $opqab$ consists of a rectangle $opqa$ and an isosceles triangle oab.

$|oa| = 18$ cm and $|ab| = |ob| = 15$ cm.

The rectangular section has sides of length 2ℓ cm and 18 cm as shown.

The centre of gravity of the lamina $opqab$ is at c, the midpoint of $[oa]$.



Taking o as the origin, find the value of ℓ.
Give your answer in the form $a\sqrt{b}$, where $a, b \in \mathbf{N}$.

(a)

$$0 = \frac{3(-x)+4(2)+1(1)+5(x)}{13}$$

$$x = -\frac{9}{2}$$ 10

$$0 = \frac{3(-3)+4(y)+1(3)+5(y)}{13}$$

$$y = \frac{2}{3}$$ 10

(b)

$$|bc| = 12 \text{ cm}$$ 5

$$\text{area of } oab = \tfrac{1}{2}(18)(12) = 108$$ 5

$$\text{c.g. of } oab = (9,\ 4)$$ 5

$$\text{area of } opqa = (18)(2\ell) = 36\ell$$ 5

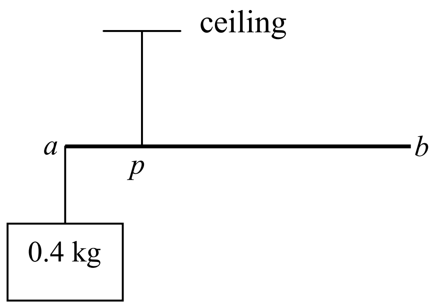
$$\text{c.g. of } opqa = (9,\ -\ell)$$ 5

$$0 = 108(4) + 36\ell(-\ell)$$

$$\ell = \sqrt{12} = 2\sqrt{3}$$ 5

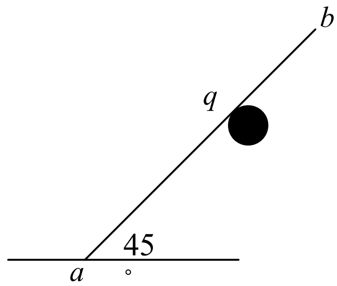
50

7. (a) A uniform rod, $[ab]$, of mass 0.1 kg and length 1 m, is suspended from a ceiling by a light, taut, inelastic string. The string is attached to the rod at the point p.
A mass 0.4 kg is attached at the end a of the rod.
The rod remains in a horizontal position and is in equilibrium.



Find $|ap|$.

(b) The uniform rod, $[ab]$, of mass 0.1 kg and length 1 m, is now placed with its end a on a smooth horizontal surface.
The rod rests on a fixed rough peg at q, where $|aq| = 0.7$ m.
The coefficient of friction between the rod and the peg is μ.
The rod is on the point of slipping when inclined at an angle 45° to the horizontal.



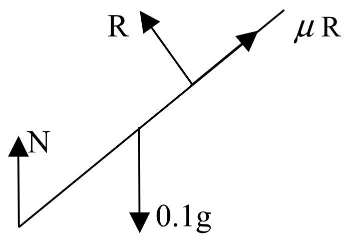
(i) Show on a diagram all the forces acting on the rod.
(ii) Find the value of μ.

(a)

$$0.4g|ap| = 0.1g(0.5 - |ap|)$$
$$|ap| = 0.1$$

20

(b) (i)



10

(ii) Moments about a:

$$R\,(0.7) = 0.1g\cos 45\,(0.5)$$
$$R = \frac{5}{7\sqrt{2}}$$

5

$$0.1g\cos 45 = R + N\sin 45$$
$$N = \frac{2}{7}$$

5

$$0.1g\sin 45 = \mu R + N\cos 45$$

5

$$\frac{1}{\sqrt{2}} = \mu\left(\frac{5}{7\sqrt{2}}\right) + \frac{2}{7\sqrt{2}}$$
$$\mu = 1$$

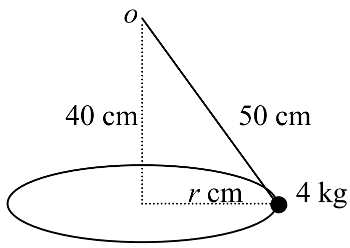
5

50

8. **(a)** A smooth particle of mass 4 kg is attached to the end of a light inextensible string 50 cm in length.
The mass describes a horizontal circle with constant speed 3 m/s on a smooth horizontal table.
The centre of the circle is also on the table.

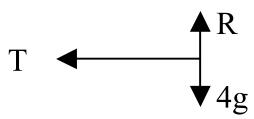
(i) Show on a diagram all the forces acting on the particle.
(ii) Find the tension in the string.

(b) A smooth particle, of mass 4 kg, describes a horizontal circle of radius *r* cm on a smooth horizontal table with constant speed 1.2 m/s.
The particle is connected by means of a light inelastic string to a fixed point *o* which is 40 cm vertically above the centre of the circle.
The length of the string is 50 cm.



(i) Find the value of *r*.
(ii) Find the tension in the string.
(iii) Find the normal reaction between the particle and the table.

(a)



$$T = \frac{mv^2}{r}$$

$$= \frac{4\,(3)^2}{0.5}$$

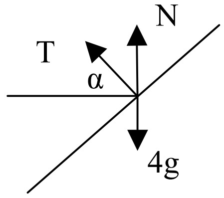
$$\Rightarrow T = 72 \text{ N}$$

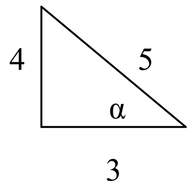
10	
5	
5	20

(b) (i) $r = \sqrt{50^2 - 40^2}$

$= 30$ cm 10

(ii)





$$T\cos\alpha = \frac{mv^2}{r}$$ 5

$$T\left(\frac{3}{5}\right) = \frac{4(1.2)^2}{0.3}$$ 5

$$T = 32 \text{ N}$$

(*iii*) $T\sin\alpha + N = 4g$ 5

$$32\left(\frac{4}{5}\right) + N = 40$$

$$N = 14.4 \text{ N}$$ 5

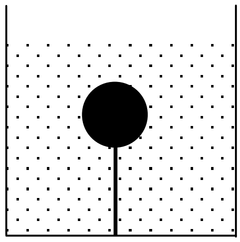
30

9. **(a)** **(i)** State the Principle of Archimedes.

A solid metal sphere of volume V m^3 has a weight of 10 newtons.
When the sphere is fully immersed in water it weighs 4 newtons.

(ii) Find the value of V.

(iii) Find the relative density of the metal.



(b) A solid metal sphere of mass 1 kg and relative density 1.5 is held immersed in a tank of liquid by a light inelastic string tied to the sphere and to the bottom of the tank.
The relative density of the liquid is 1.8.

(i) Show, on a diagram, all the forces acting on the sphere.

(ii) Find the tension in the string.

The string is removed and the sphere is taken out of the tank of liquid. The sphere is now placed into a tank of water so that it rests, fully immersed, on the bottom of the tank.

(iii) Find the normal reaction between the bottom of the tank and the sphere.

Give your answer in the form $\frac{a}{b}$, where $a,b \in \mathbf{N}$.

(a)

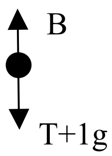
(i) : Principle of Archimedes 5

(ii)
$$B = \text{weight of liquid displaced}$$
$$B = \rho V g \quad 5$$
$$10 - 4 = 1000(V)(10)$$
$$V = 0.0006 \text{ m}^3 \quad 5$$

(iii)
$$\text{density} = \frac{\text{mass}}{\text{volume}} = \frac{1}{0.0006} \quad 5$$
$$\text{relative density} = \frac{1}{0.6} = \frac{10}{6} \text{ or } \frac{5}{3} \text{ or } 1.7 \quad 5$$

(b)

(i)



10

(ii)
$$T + 10 = \frac{W_I \, s_L}{s} \quad 5$$
$$T + 10 = \frac{10\,(1.8)}{1.5}$$
$$T = 2 \quad 5$$

(iii)
$$N + \frac{W_I \, s_L}{s} = 10$$
$$N + \frac{10\,(1)}{1.5} = 10$$
$$N = \frac{10}{3} \quad 5$$

50